A Sensitive Probe for Measuring Concentration Profiles in Water Vapor-Gaseous Mixtures

MAILAND R. STRUNK, MILAN V. MITROVIC, and DAVID W. BUNCH

University of Missouri School of Mines and Metallurgy, Rolla, Missouri

Although the use of lithium chloride in measuring the amount of water vapor in air is not new, it is the simplicity of construction of such a probe, its ease of calibration, its accuracy and great utility in mass transfer studies that form the basis for this note. A small concentration probe, capable of measuring accurately the concentration of water vapor in air or other nonreactive gases at a point or at different radial positions in a flowing stream, can be constructed and effectively utilized. All measurements made with such a probe indicate that excellent concentration profiles may be obtained directly in a manner similar to that for obtaining velocity or temperature profiles. The principle involved is essentially one of equilibrium between water vapor and the lithium chloride on the probe under constant temperature conditions. In a flowing stream at steady state conditions equilibrium is established in about 2 min. and, in the case of a wetted-wall column, the varying concentrations of water vapor at the various radial positions affect the resistance of the lithium-chloride probe. Dunmore (*1*) developed devices using lithium chloride to measure relative humidity. Although devices similar to his are discussed throughout the literature (*2* to *10*) the adoption of this technique for measuring concentration profiles in mass transfer experiments has not been fully exploited. Such a probe easily may be constructed and calibrated and made applicable over wide ranges of water vapor concentrations.

The probe can be constructed from any thin glass tubing or other suitable material. Typical dimensions of the tube used by the authors were approximately 1 mm. in diam. and about 3 in. in length. A thin silver or platinum wire, about 0.1 mm. in diam., is fed through the inside of the glass tubing. Enough wire is fed through the tubing to provide sufficient length for wrapping the wire in a helix around the outside surface of the glass tube. The end of the wire, after forming the helix, may be cemented to the glass surface. A second wire or lead is cemented to the outside surface of the glass and also wrapped in a helix parallel to the first wire. The end of this wire may be

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Milan V. Mitrovic is at the University of Belgrade, Belgrade, Yugoslavia.

A Note on the Prediction of Wall Shear Stress Distribution in the Eccentric Annulus

W. T. SNYDER

Brookhaven National Laboratory, Upton, New York

In applying a generalized velocity correlation to the calculation of turbulent velocity fields, a knowledge of the wall stress is generally required. For example, in the Deissler-Taylor method (*1*) the wall stress must be found by a laborious iterative procedure for irregular passages such as eccentric annuli.

The interest in the flow and heat transfer characteristics of eccentric annuli stems from the utility of the eccentric annulus as a model of the misalignment of a tube in an otherwise symmetric tube or rod bundle with in-line flow. Experimental data taken on different rods in the same tube bundle indicate that the problem of tube misalignment may be a serious one with drastic changes resulting in the heat transfer rate (*2*). A calculation of the heat transfer coefficient for the eccentric annulus requires a knowledge of the velocity distribution. The purpose of the present note is to illustrate a simple method for predicting the variation of wall stress around the inner and outer surfaces of an eccentric annulus.

THE ANALYSIS

The geometry of the eccentric annulus is illustrated in Figure 1. The major assumption involved in the present analysis is that the local wall stress in the eccentric annulus may be determined by treating each local wall position as a position in an effective concentric annulus in a manner to be subsequently explained.

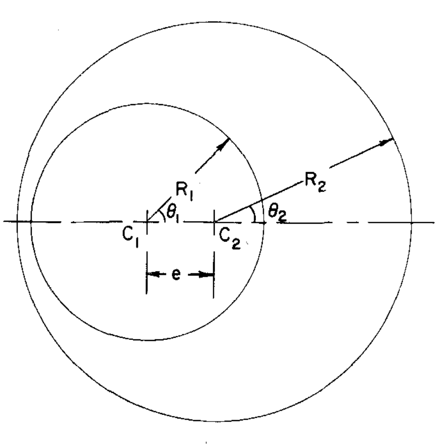

Fig. 1. Eccentric annulus geometry.

In a concentric annulus with inner and outer radii of r_1 and r_2 respectively, the locus of maximum velocity occurs at some radius r_m between r_1 and r_2. For laminar flow, the radius of maximum velocity is given by the equivalent expressions

$$(r_m/r_1)^2 = \frac{(r_2/r_1)^2 - 1}{2 \ln (r_2/r_1)} \quad (1a)$$

and

$$(r_m/r_2)^2 = \frac{(r_1/r_2)^2 - 1}{2 \ln (r_1/r_2)} \quad (1b)$$

It has been verified experimentally that Equation (1) is also valid for turbulent flow (*3*).

Because the radius r_m is a line of zero shear stress, a force balance on the

W. T. Snyder is at the State University of New York, Stony Brook, New York.

element of fluid between r_1 and r_m gives

$$\tau_1 = \frac{r_1}{2}\frac{dp}{dz}\{(r_m/r_1)^2 - 1\} \qquad (2)$$

where τ_1 is the wall stress at r_1 and $\frac{dp}{dz}$ is the pressure gradient. A similar expression may be written for the wall stress at r_2, namely

$$\tau_2 = \frac{r_2}{2}\frac{dp}{dz}\{1 - (r_m/r_2)^2\} \qquad (3)$$

In the present analysis, Equations (2) and (3) are assumed to apply locally along R_1 and R_2, respectively. In contrast to the concentric case, r_m is not a constant in the eccentric annulus but varies with position. The procedure for determining r_m is as follows.

If τ_1 is being calculated, the center C_1 shown in Figure 1 will be taken as the center of the effective concentric annulus, the radius r_1 is R_1, and the radius r_2 is the distance along a radial line from C_1 to the outer surface for a fixed value of θ_1. Applying the law of cosines gives

$$R_2^2 = r_2^2 + e^2 - 2er_2\cos\theta_1 \qquad (4)$$

After some rearrangement, Equation (4) may be written

$$\left(\frac{r_2}{r_1}\right)_1 = \phi\left(\frac{1}{\gamma} - 1\right)\cos\theta_1 + \sqrt{\left\{\phi\left(\frac{1}{\gamma} - 1\right)\cos\theta_1\right\}^2 + \frac{1}{\gamma^2}(1-\phi^2) + \phi^2\left(\frac{2}{\gamma} - 1\right)} \qquad (5)$$

where

$$\gamma = R_1/R_2 \qquad (5a)$$

$$\phi = \frac{e}{R_2 - R_1} \qquad (5b)$$

The average stress along r_1 is given by the expression

$$\bar{\tau}_1 = \frac{1}{\pi}\int_0^{\pi} \tau_1 d\theta_1 \qquad (6)$$

A combination of Equations (2) and (6) gives

$$\tau_1/\bar{\tau}_1 = \frac{(r_m/r_1)^2 - 1}{\frac{1}{\pi}\int_0^{\pi}(r_m/r_1)^2 d\theta_1 - 1} \qquad (7)$$

The quantity (r_m/r_1) in Equation (7) is to be determined by Equation (1*a*) with r_2/r_1 given by Equation (5).

To determine the distribution of τ_2, C_2 is taken as the center of the effective concentric annulus, r_2 becomes R_2, and the radius r_1 is the distance along a radial line from C_2 to the inner surface for a given value of θ_2. When the law of cosines is applied

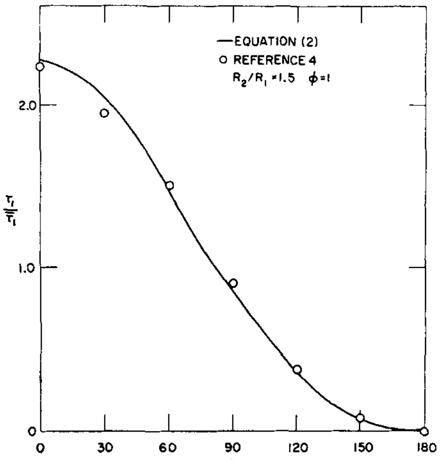

Fig. 2. Inner wall shear stress distribution.

$$R_1^2 = r_1^2 + e^2 - 2er_1\cos(\pi - \theta_2) \qquad (8)$$

and

$$\left(\frac{r_1}{r_2}\right)_2 = -\phi(1-\gamma)\cos\theta_2 + \sqrt{\{\phi(1-\gamma)\cos\theta_2\}^2 + \gamma^2(1-\phi^2) + \phi^2(2\gamma - 1)} \qquad (9)$$

The expression for the distribution of wall stress then becomes

$$\tau_2/\bar{\tau}_2 = \frac{1-(r_m/r_2)^2}{1 - \frac{1}{\pi}\int_0^{\pi}(r_m/r_2)^2 d\theta_2} \qquad (10)$$

The ratio (r_m/r_2) in Equation (10) is to be determined by Equation (1*b*) with r_1/r_2 given by Equation (9).

To determine $\bar{\tau}_1$ and $\bar{\tau}_2$ for a given pressure gradient, force balance equations may be written for the fluid between R_1 and r_m and between r_m and R_2. When the equations are written for

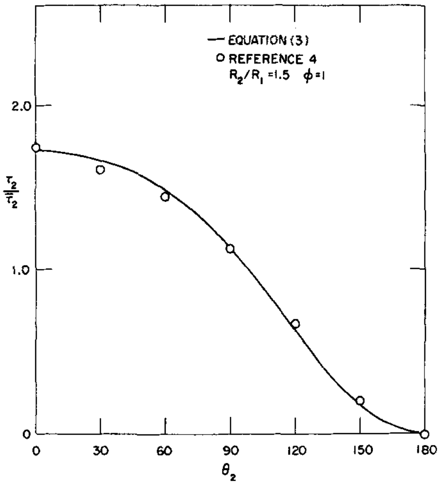

Fig. 3. Outer wall shear stress distribution.

the separate portions of the cross section

$$\pi R_1 \bar{\tau}_1 = \frac{dp}{dz} A_1 \qquad (11)$$

$$\pi R_2 \bar{\tau}_2 = \frac{dp}{dz} A_2 \qquad (12)$$

where A_1 is the cross-sectional area included between R_1 and r_m given by Equations (1*a*) and (5), and A_2 is the cross-sectional area included between R_2 and the r_m given by Equations (1*b*) and (9).

In reference *4* a solution for the velocity distribution is obtained by the Deissler-Taylor method (*1*) for an eccentric annulus with $R_2/R_1 = 1.5$ and $\phi = 1$. From this solution, values of wall stress were determined for both inner and outer surfaces. In Figures 2 and 3, a comparison between the shear stress distribution from reference *4* and the distribution based on the present analysis is shown. The agreement is seen to be quite satisfactory.

NOTATION

C_1	=	center of inner circle
C_2	=	center of outer circle
e	=	eccentricity
r_1	=	inner radius of concentric annulus
r_2	=	outer radius of concentric annulus
r_m	=	radius of maximum velocity for concentric annulus
R_1	=	inner radius of eccentric annulus
R_2	=	outer radius of eccentric annulus
$\frac{dp}{dz}$	=	pressure gradient
θ_1, θ_2	=	angular positions shown in Figure 1
γ	=	R_1/R_2
ϕ	=	eccentricity ratio, $e/(R_2 - R_1)$
τ_1, τ_2	=	inner and outer wall stresses, respectively
—	=	average quantity

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